APPENDIX NOTES

NOTE 1: For the convenience of users of this standard, excerpts of the original journal articles and proceedings papers on which definitions are based are reproduced here with permission. In some cases minor changes were made for improved clarity, including replacing the word average (which could be the mean, median, or mode) with the word mean where appropriate.

NOTE 2: Many of the original articles and papers did not include lists of symbols. To be consistent with SAE practice, they have been added at the end of each appendix, using the original notation. There may be cases where a particular variable is represented using different symbols in two appendices to maintain consistency with the original source material and assure accurate representation. As a result, variables used in each appendix are local to that appendix.

APPENDIX A – CALCULATION OF TIME TO COLLISION (TTC)

A.1 BACKGROUND

Time to Collision (TTC), as defined in section 7.3.1, is the duration, usually measured in seconds, required for one vehicle to strike another object. There are two versions: Option A that includes acceleration and velocity, and Option B that considers only velocity. In general, the larger the value of TTC, the safer the driver is. It can be applied to a single driver, a specified user class, or all vehicles that pass a given road segment during a time period.

A.2 COMPUTATIONAL METHOD

The method for calculating TTC is taken almost verbatim from van der Horst (1990), p. 167–170.

A.2.1 Procedure

Let the situation at a given time t = T be given by Figure A1.

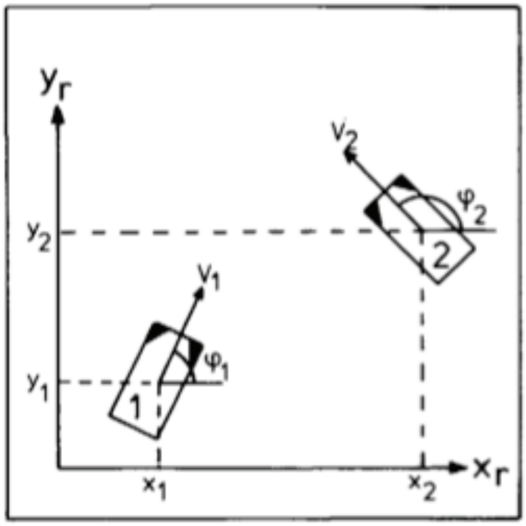

FIGURE A1 - SITUATION AT T = T WITH x_i = X-COORDINATE IN THE ROAD PLANE, y_i = Y-COORDINATE, v_i = SPEED,
a_i = acceleration and ϕ_i = heading angle of vehicle *i* (*i* = 1, 2).

Irrespective of which assumption is made for the continuation of movement from the moment T on, the TTC concept always requires two steps:

1. detect whether both vehicles have a mutual collision course, and if so,
2. calculate TTC at moment t = T.

When two road users are approaching each other, in general, there will be an area of intersection S, defined by the dimensions of the vehicles (or the vehicle and the object). An example for the simple case of a perpendicular angle of intersection is given in Figure A2.

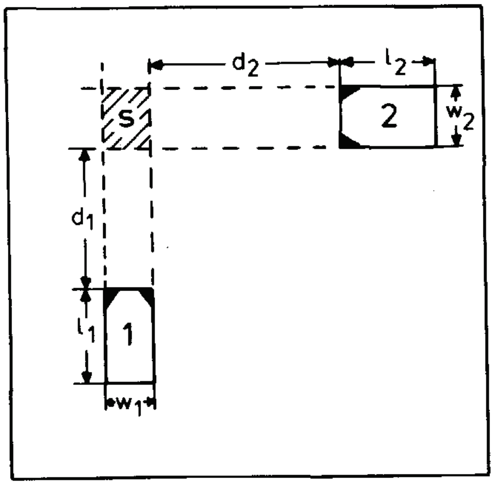

FIGURE A2 - SITUATION AT T = T FOR A PERPENDICULAR APPROACH WITH THE AREA OF INTERSECTION = S, l_i = LENGTH, w_i = WIDTH OF VEHICLE *i* (*i* = 1, 2) AND d_i = DISTANCE FROM FRONT OF VEHICLE *i* TO AREA S.

A.2.2 Determination of Collision

A collision course will only occur if one of the following conditions is satisfied:

$$t_{f1} < t_{f2} < t_{r1} \quad \text{(A1)}$$

or

$$t_{f2} < t_{f1} < t_{r2} \quad \text{(A2)}$$

where t_{f1}, t_{f2} = the moment the fronts of vehicle 1 and vehicle 2, respectively, reach area S, and
t_{r1}, t_{r2} = the moment the rear of vehicle 1 and vehicle 2, respectively, leaves area S.

If neither Eq. A1 nor Eq. A2 is satisfied, there is no collision course, and consequently, TTC will be infinite.

If Eq. (A1) is true, the TTC value at time t = T is given by:

$$\text{TTC} = t_{f2} - \text{T} \quad \text{(A3)}$$

while if Eq. (A2) is true:

$$\text{TTC} = t_{f1} - \text{T} \quad \text{(A4)}$$

Of course, t_{f1} through t_{f2} will depend on the positions, the speeds, and the heading angles at t = T, as well as on the assumption of the continuation of movements from time T on.

A.2.3 TTC Based On Constant Speed And Heading Angle, Perpendicular Approach

If the continuation or movement is defined by a constant remaining speed and heading-angle, the time moments for the example of Figure A2 are given by:

$$t_{f1} = \text{T} + d_1 / v_1 \quad \text{(A5)}$$

$$t_{r1} = \text{T} + (d_1 + l_1 + w_2) / v_1 \quad \text{(A6)}$$

$$t_{f2} = \text{T} + d_2 / v_2 \quad \text{(A7)}$$

$$t_{r2} = \text{T} + (d_2 + l_2 + w_1) / v_2 \quad \text{(A8)}$$

If Eq. (A1) is satisfied, then substituting Eq. (A7) into Eq. (A3) gives:

$$\text{TTC} = d_2 / v_2 \quad \text{(A9)}$$

And, if Eq. (A2) is satisfied, then substituting Eq. (A5) in Eq. (A4) gives:

$$\text{TTC} = d_1 / v_1 \qquad \text{(A10)}$$

A.2.4 Non-Perpendicular Approaches

A.2.4.1 General Considerations

For non-perpendicular angles of intersection, all corner points of both vehicles have to be considered separately to determine whether a collision course is present or not. Also, more types or potential collisions have to be taken into account. For an acute angle, for example, six different collision types are possible (Figure A3) with separate conditions and equations for collision course and calculation of TTC.

In addition, both rear-end and head-on approaches require different computations.

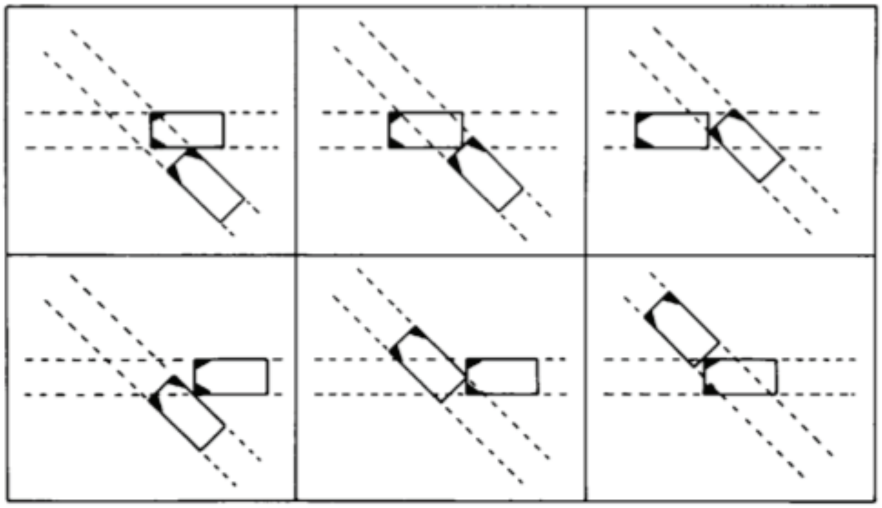

FIGURE A3 - TYPES OF POTENTIAL COLLISIONS FOR AN ACUTE ANGLE OF APPROACH

A.2.4.2 TTCA (TTC Based On Constant Acceleration And Heading Angle)

When the continuation of movement is based on constant remaining accelerations (or decelerations), mathematical expressions for t_{f1} through t_{r2} can also be derived, with some restrictions dealing with potential stops before or after entering the area S. However, a more general computational approach can be followed by using numerical methods for solving a set of higher order equations, with both x and y from time T on given as an nth order polynomial in t.

One example, called adjusted TTC in Appendix C, uses the mean accelerations of the lead and following vehicles from brake onset to collision to calculate the minimum TTC value.

A.3 LIST OF SYMBOLS

Symbol	Definition	Units
t	time	s
T	specific moment of time	s
a_i	acceleration	m/ s^2
ϕ_i	heading angle of vehicle i, where i is the vehicle number (1, 2)	deg
x_i	X coordinate	m
y_i	Y coordinate	m
v_i	speed	m/s
TTC	Time to Collision	s
S	area of intersection	m^2
l_i	length of vehicle i	m
w_i	width of vehicle i	m
d_i	distance from front of vehicle i to area S	m
TTCA	TTC based on constant acceleration and heading angle	s

APPENDIX B – CALCULATION OF MINIMUM TIME TO COLLISION (TTC_{min})

B.1 BACKGROUND

Larger values of minimum time to collision (TTCmin) indicate a greater margin of safety. TTCmin as defined in section 9.3.6 is the minimum duration required for one vehicle to strike another vehicle or object over some time period on the order of seconds.

B.2 COMPUTATIONAL METHOD

The method for calculating minimum TTC is taken almost verbatim from van der Horst (1990) p. 170–171.

The minimum TTC when two road users are on a collision course at a certain moment is determined by comparing all TTC values that are present in the encounter. No separate computation is needed to obtain $TTC_{\min}$.

To illustrate how $TTC_{\min}$ depends on acceleration, consider the simple example of a car having a constant acceleration approaching a fixed object. At time t = 0 the constant acceleration (deceleration) is instantaneously in effect. For a movement with a constant acceleration, the following equations hold:

Acceleration: $a(t) = -a \quad \text{m}/s^2$ (B1)

Speed: $v = v_0 + at$ (B2)

Distance travelled: $d = v_0 t + 0.5a t^2$ (B3)

where v_0 is initial velocity at t=0

d_0 is the initial distance to the fixed object at t=0.

The TTC at moment t is given by:

$$TTC = (d_0 - d)/v \qquad \text{(B4)}$$

Substituting equations B2 and B3 in equation B4 gives:

$$TTC = (d_0 - v_0 t - 0.5a t^2) / (v_0 + at) \qquad \text{(B5)}$$

The moment the minimum of TTC is reached ($t = t_{\min}$) is determined when the derivative of TTC equals zero:

$$d/dt(TTC) = 0,$$

or

$$\{(v_0 + at)(-v_0 - at) - (d_0 - v_0 t - 0.5a t^2)\} / (v_0 + at)^2 = 0$$

resulting in:

$$0.5\, a^2 t^2 + a v_0 t + v_0^2 + d_0 a = 0 \qquad \text{(B6)}$$

The condition of no collision is only valid if equations B7 and B8 are both satisfied, i.e.:

$$-a \geq v_0^2/(2 \cdot d_0) \qquad \text{(B7)}$$

and

$$t < -a/v_0 \qquad \text{(B8)}$$

resulting in:

$$t_{\min} = -v_0/a + (-v_0^2 - 2 d_0 a)^{1/2}/a \qquad \text{(B9)}$$

Substituting t = $t_{\min}$ in equation B5 gives:

$$TTC_{\min} = (d_0 - v_0 t_{\min} - 0.5a\, t_{\min}^2) / (v_0 + a\, t_{\min}) \qquad \text{(B10)}$$

In Eq. B10, $TTC_{\min}$ is given as a function of the acceleration (a) and the distance and speed at moment t = 0. But $TTC_{\min}$ can also be expressed as a function of the speed and distance at moment $t = t_{\min}$, viz.:

$$TTC_{\min} = d_{\min} / v_{\min} \tag{B11}$$

Since it can easily be derived, that $d_{\min}$ also equals $-v_{\min}^{2}$/a, equation B11 results in:

$$TTC_{\min} = -v_{\min} / a \tag{B12}$$

The time it takes to come to a stop from moment $t_{\min}$ on, also equals $-v_{\min}$ / a. This implies that at time $t_{\min}$, $TTC_{\min}$ is equal to the time it takes from the current moment to come to a complete stop. From this, a simple decision rule may be derived, viz. if TTC is less than the remaining stopping time, continue braking. After TTC reached its minimum, TTC will be greater than the remaining stopping time, implying that the deceleration may decrease.

B.3 LIST OF SYMBOLS

Symbol	Definition	Units
a	acceleration	m/s^2
d	distance travelled	m
d_{min}	distance travelled at time t_{min}	m
TTC	Time-To-Collision	s
$TTC_{\min}$	minimum TTC	s
V	speed	m/s
$V_{t\min}$	speed at $t_{\min}$	m/s

APPENDIX C – CALCULATION OF MINIMUM ADJUSTED TIME TO COLLISION

C.1 BACKGROUND

Adjusted time to collision as defined in section 7.3.3 "is the amount of spare time the driver had based on the avoidance response chosen by the driver. Positive values indicate the amount of extra time the driver had based on the deceleration profile. Negative values indicate how much earlier the driver would have needed to begin the response in order to have avoided the collision" (Brown, 2005, p. 42). Adjusted TLC takes into account the relative velocity at time of collision, and the mean accelerations of the lead and following vehicles.

C.2 COMPUTATIONAL METHOD

The calculation method shown here is almost verbatim from Brown (2005), p. 42-43. Brown computes adjusted minimum TTC using the acceleration-based TTC value (Option A, section 7.3.1.1), which he refers to as Type II TTC. However, a velocity-based TTC value (Option B, section 7.3.1.2) could also be used.

C.2.1 Procedure

C.2.1.1 No collision

In the case of no collision, minimum adjusted TTC is the minimum value of TTC. When both vehicles are moving and the lead vehicle is decelerating, TTC is derived from the following equation of motion assuming continued travel at the current speed by the driver's vehicle.

$$-R = \frac{1}{2}\alpha \times TTC^2 + \dot{R} \times TTC \qquad \text{(C1)}$$

where

R = Range

$\dot{R}$ = Lead Vehicle Velocity – Following Vehicle Velocity

α = Lead Vehicle Acceleration

TTC is then derived using the quadratic formula as follows:

$$TTC = -\frac{\dot{R} + \sqrt{(\dot{R})^2 - (2\alpha)(R)}}{\alpha} \qquad \text{(C2)}$$

Using the same definition of range rate, when the lead vehicle is stationary or travelling at a constant speed, TTC is simply a function of range and range rate expressed as follows:

$$TTC = R/-\dot{R} \qquad \text{(C3)}$$

The above calculations for minimum adjusted TTC would result in a value of zero in the case where a collision occurs, regardless of whether the differential velocity between the two vehicles at the time of collision is very small or very large. As a result, minimum TTC is restricted in range to non-negative values, and the distribution of TTC_{min} becomes non-normal as more crashes occur in the dataset.

C.2.1.2 Collision

To calculate the adjusted minimum TTC in the case of a crash, the situation preceding the crash is considered. If the lead vehicle is stopped:

$$\text{Adjusted Minimum TTC} = V_F/\bar{a}_F \qquad \text{(C4)}$$

where:

V_F = Following Vehicle Velocity at the Time of Collision

$\bar{a}_F$ = Mean Acceleration of the Driver's Vehicle from Brake onset to collision

If the lead vehicle is moving and the following vehicle is decelerating as quickly as the lead vehicle or greater:

$$\text{Adjusted Minimum TTC} = (V_F - V_L)/(\bar{a}_F - \bar{a}_L) \tag{C5}$$

where:

V_F = Following Vehicle Velocity at the Time of Collision

V_L = Lead Vehicle Velocity at the Time of Collision

$\bar{a}_F$ = Mean Acceleration of the Following Vehicle from Brake onset to collision

$\bar{a}_L$ = Mean Acceleration of the Lead Vehicle from Brake onset to collision

By definition, if the lead vehicle is moving and the following vehicle is not decelerating as quickly as the lead vehicle, the driver could not have avoided the collision based on the current response, and:

$$\text{Adjusted Minimum TTC} = -\infty \tag{C6}$$

C.2.2 Example

An example provided in Brown (2005) and summarized here illustrates the differences between TTC_{min} and adjusted TTC_{min}. Brown simulated two scenarios differing primarily in their assumed deceleration levels in order to obtain different mixtures of crashes and non-crashes. Figure C1 shows normal probability plots for TTC_{min}, relative velocity at collision, and adjusted TTC_{min} for the two scenarios. For the top row of plots (deceleration = 0.4 g), only 2 collisions occurred, resulting in very little difference between TTC_{min} and adjusted TTC_{min}. For the bottom row of plots (deceleration = 0.75 g) there were many more collisions (a total of 15), resulting in significant differences between TTC_{min} and adjusted TTC_{min}. The adjusted TTC_{min} values are normally distributed, enabling use of parametric statistics to analyze them.

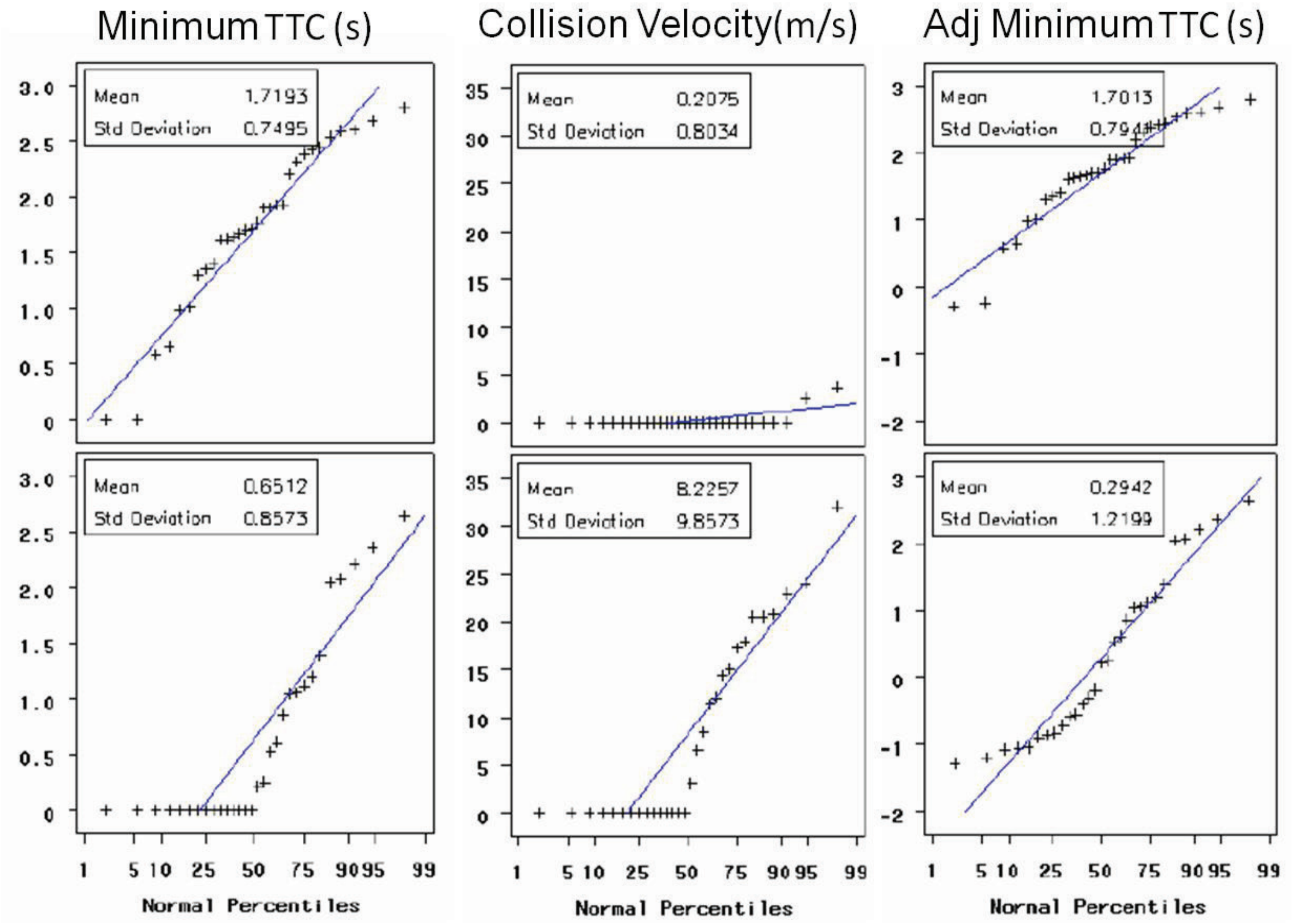

FIGURE C1 - NORMAL PROBABILITY PLOTS FOR TTC$_{min}$, RELATIVE VELOCITY AT COLLISION, AND ADJUSTED TTC$_{min}$ FOR ASSUMED DECLERATIONS OF 0.4 G (TOP ROW) AND 0.75 G (BOTTOM ROW)
Source: Brown (2005), p. 46

C.3 List of Symbols

Symbol	Definition	Units
TTC	Time to Collision	s
TTC$_{min}$	Minimum Time to Collision (for crashes, TTC$_{min}$ = 0)	s
Adjusted TTC$_{min}$	Adjusted Minimum Time to Collision (for crashes, Adjusted TTC$_{min}$ < 0)	s
R	range	m
$\dot{R}$	lead vehicle velocity – following vehicle velocity	m/s
a	lead vehicle acceleration	m/s^2
V_F	following vehicle velocity at the time of collision	m/s
$\bar{a}_F$	mean acceleration of the following vehicle from brake onset to collision	m/s^2
V_L	lead vehicle velocity at the time of collision	m/s
$\bar{a}_L$	mean acceleration of the lead vehicle from brake onset to collision	m/s^2

APPENDIX D - CALCULATION OF TIME EXPOSED TIME TO COLLISION (TET)

D.1 BACKGROUND

Time Exposed to Collision (TET), as defined in section 7.3.4, is the duration of time over which the time to collision measure is below some undesired threshold. TET is a more safety-relevant measure than TTC alone because it considers exposure time. It can be applied to a single driver, a specified user class, or all vehicles that pass the road segment during a time period, and can distinguish impacts per lane. The original definition in the literature does not indicate if acceleration is considered, though the definitions provided in Appendix A do.

D.2 COMPUTATIONAL METHOD

The calculation method shown here is almost verbatim from Minderhoud and Bovy (2001), p. 92-94.

D.2.1 Procedure

Calculation of Time Exposed Time-to-collision (TET) requires collection of the position and speed of all vehicles entering and leaving a road section bounded by X_1 and X_2, over time period H, from which trajectories and time-to-collision profiles (Figure D1) can be established.

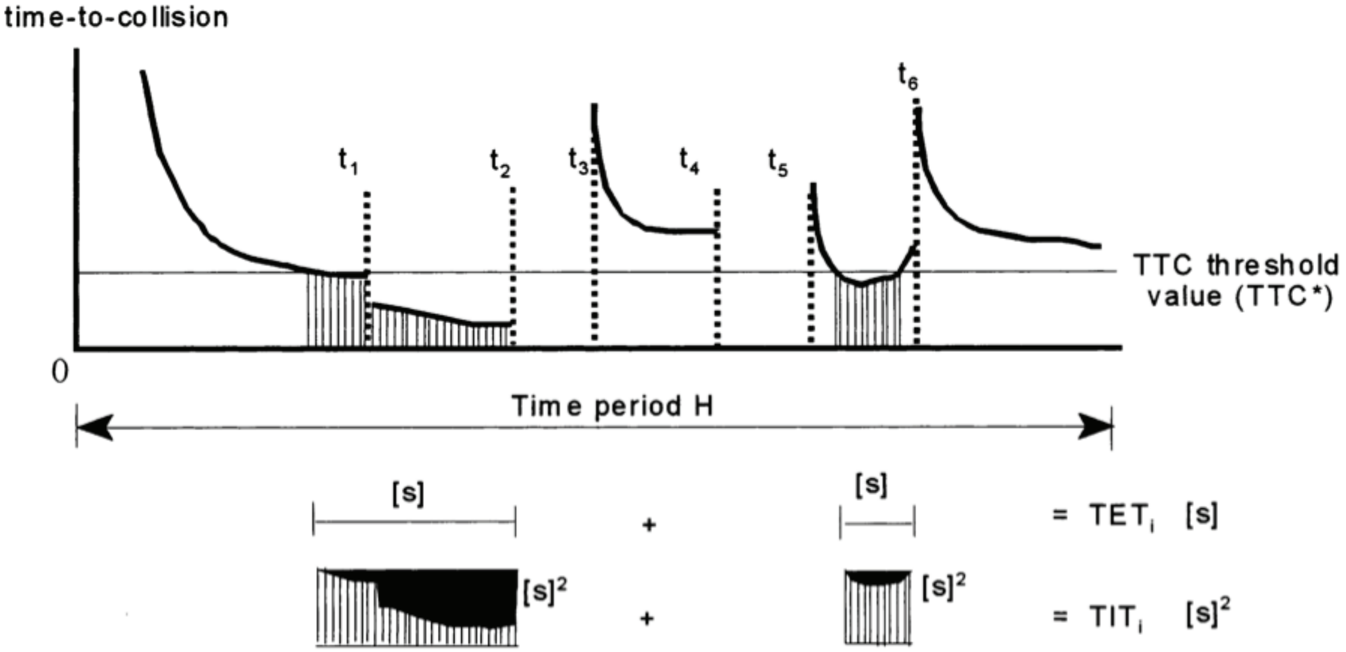

FIGURE D1 - EXAMPLE TIME-TO-COLLISION PROFILE OF A DRIVER-VEHICLE COMBINATION I IN MOTORWAY TRAFFIC (VERTICALLY SHADED AREAS REPRESENT SAFETY-CRITICAL APPROACH CONDITIONS). ADAPTED FROM MINDERHOUD AND BOVY (2001), P. 92.

One assumption is that the measured TTC values at any instant t do not change during a small time step τ_{sc} (e.g., 0.1 s), the spacing between vertical lines in Figure D1. Over the time period H there are $T = H/\tau_{sc}$ time instants t (t = 0, 1, 2,…., T) to calculate TTC values.

TET a summation of all moments (over the time period H) that a driver approaches a front vehicle with a TTC-value below the threshold value TTC*, the latter is considered to be the boundary between safe and safety-critical approaches. Thus, the lower the TET value, the more safe the situation (on average over period H). This safety measure does not take into account the variation in safety levels of different time-to-collision values below the threshold value.

The TET* value (seconds) for a driver/vehicle i can be expressed as follows

$$TET_i^* = \sum_{t=0}^{T} \delta_i(t) \cdot \tau_{sc} \qquad \text{(D1)}$$

where δ is an indicator variable defined as

$$\delta_i(t) = \begin{cases} 0 & \text{else} \\ 1 & \forall\ 0 \leq \mathrm{TTC}_i(t) \leq \mathrm{TTC}^* \end{cases}$$

When driver *i* at instant t experiences a TTC-value between 0 and the specified threshold value TTC*, the value of δ is 1. Otherwise, the value of δ is 0.

The total TET* for a population of N vehicles (drivers) can be expressed as

$$TET^* = \sum_{i=1}^{N} TET_i^* \qquad \text{(D2)}$$

Note – The superscript * indicates the TET value is calculated with respect to the threshold TTC value.

A TET value can also be calculated separately per user class, e.g. trucks and passenger cars, or vehicles equipped and not equipped with intelligent driver support systems, by adding an extra index and summation per user class.

D.2.2 Summary Values For TET

A mean TET value per vehicle, TET*, can be computed as

$$\text{Mean TET*} = \text{TET*}/\text{N} \qquad \text{(s/vehicle)} \qquad \text{(D3)}$$

in order to standardize TET across sample size and duration of the observations.

The mean value still includes the time period over which the TET value has been determined. To overcome this dependency, an indicator *P** can be established, expressing the probability that a vehicle encounters a safety-critical approach situation, which is defined as a moment with a TTC-value between 0 and TTC* seconds. The TET* probability per vehicle is calculated by dividing the mean indicator value of Eq. D3 by the maximum attainable time period H.

$$\text{TET } P^* = 100\ (\text{Mean TET*}) / \text{H} \quad (\%) \qquad \text{(D4)}$$

The probability indicator can be interpreted as the percentage of time that a random driver on average drives with TTC values below the threshold TTC*.

D.2.3 Threshold Values For TTC*

A three-second threshold TTC* is considered an adequate level for discriminating dangerous approach situations from acceptable situations, as has been observed by Hirst and Graham (1997). Nevertheless, other TTC-threshold values can be used.

D.3 LIST OF SYMBOLS

Symbol	Definition	Units
TTC	Time to Collision	s
TTC*	Time to Collision threshold value	s
TET	Time Exposed Time-to-collision	s
TET*	TET value for a population of N vehicles	s
P^*	probability that a vehicle encounters a safety-critical approach situation (a moment with a TTC-value between 0 and TTC* seconds)	%
t	time	s
δ	indicator variable	
N	number of vehicles (or drivers)	
H	period	s
T	number of time steps in H	
τ_{sc}	small time step	s

APPENEDIX E - CALCULATION OF TIME INTEGRATED TIME TO COLLISION (TIT)

E.1 BACKGROUND

Time Integrated Time to Collision (TIT), as defined in section 7.3.5 is the duration over which the time to collision is below some undesired threshold weighted by how far below that threshold the time to collision is at each moment. One disadvantage of the TET measure is that a TTC-value much less than the threshold TTC value does not have a greater affect on the TET value. However, it may be expected that an extremely small TTC-value (e.g. smaller than 0.5 s) represents an approach situation with a relatively high probability of a collision compared to greater TTC-values (e.g. 2 or 3 s). For that reason, the Time Integrated Time-to-collision (TIT) measure has been developed.

E.2 COMPUTATIONAL METHOD

The method for calculating TIT is almost verbatim from Minderhoud and Bovy (2001), p. 92-94.

E.2.1 Procedure

E.2.1.1 Continuous time interval

In continuous time, the TIT indicator measure (TIT*) for a population of vehicles is the integral of the time-to-collision profile over time periods when the TTC value is below the threshold value.

$$TIT^* = \sum_{i=1}^{N} \int_0^T [TTC^* - TTC_i(t)] dt \qquad \forall 0 \le TTC_i(t) \le TTC^* \qquad \text{(E1)}$$

The vertically shaded areas in Figure D1 represent situations in which the driver approaches the front vehicle with TTC-values below TTC*. Since low TTC-values represent more dangerous situations, it holds that the smaller the shaded area, the higher the risks of collisions. To be consistent with the TET-indicator, the shaded area should be subtracted from the area below the threshold value, resulting in a time integral with an interpretable meaning. This area is shown in Figure D1 by a dark surface. A high TIT value means a greater exposure to less safe TTC values.

E.2.1.1 Discrete Time Intervals

The individual TIT for driver/vehicle i in discrete time can be calculated with:

$$TIT_i^* = \sum_{t=0}^{T} [TTC^* - TTC_i(t)] \cdot \tau_{SC} \qquad \forall 0 \le TTC_i(t) \le TTC^* \qquad \text{(E2)}$$

Summation over all vehicles (i = 1…N) present in the road section of interest during time period H, results in the following discrete-time aggregate TIT (in s^2)

$$TIT^* = \sum_{i=1}^{N} TIT_i^* \qquad \text{(E3)}$$

The mean duration that a vehicle encounters an unsafe situation is

$$\text{Mean TIT}^* = \text{TIT}^* / \text{N} \qquad (s^2/\text{vehicle}) \qquad \text{(E4)}$$

The TIT* probability indicator can be calculated by dividing the mean TIT* in Eq. E4 by the theoretically maximum attainable TIT value per vehicle (H • TTC*).

$$\text{TIT } P^* = 100 \, (\text{Mean TIT}^*) / (\text{TTC}^* \bullet \text{H}) \qquad (\%) \qquad \text{(E5)}$$

E.3 LIST OF SYMBOLS

Symbol	Definition	Units
TIT	Time Integrated Time-to-collision	s^2
TIT*	TIT indicator value for a population of N vehicles	s^2
t	time	s
TTC	Time to Collision	s
TTC*	Time to Collision threshold value	s
TET	Time Exposed Time-to-collision	s
N	number of vehicles	
*P**	probability that a vehicle encounters a safety-critical approach situation (a moment with a TTC-value between 0 and TTC* seconds)	%
H	time period	s
T	number of time steps in H	

APPENDIX F – CALCULATION OF STEERING WHEEL REVERSALS

F.1 BACKGROUND

When drivers are paying attention to driving, they make a large number of small steering corrections. However, when distracted, corrections may be fewer, but often are larger. Thus, the pattern of reversals changes when distracted. Typically, a correction involves turning the steering wheel to change the yaw angle and lateral position of the vehicle and subsequently turning the steering wheel a second time in the opposite direction to change the yaw angle again so the vehicle heads straight.

Steering Wheel Reversals, as defined in section 8.3.1, occur when a steering wheel rotates at least Δa deg in one direction and then rotates at least Δa deg in the opposite direction within a moving time window Δt.

F.2 COMPUTATIONAL METHOD

The method shown here for calculating steering wheel reversals is reproduced verbatim from the AIDE report (Ostlund, Peters, Thorslund, Engstrom, Markkula, Keinath, Horst, Juch, Mattes, and Foehl, AIDE D2.2.5, 2005, page 126-128).

The steps described below are applied to the steering wheel angle signal.

F.2.1 Apply Low-Pass Filter

A low-pass second-order Butterworth filter with cut-off frequency 0.6 *Hz* is applied. The filter reduces high-frequency noise in the steering wheel angle signal, and makes it possible to find stationary points (local maxima and minima) using the method described below.

F.2.2 Find Stationary Points

Let Θ_i be the value of the low-pass filtered steering wheel angle signal at time step *i*, with *i* = {1, 2, 3,…, *T*}, where *T* is the total number of samples in a measurement. Calculate the following quantity:

$$\theta'_i = \begin{cases} 0 & i = 1 \\ \theta_i - \theta_{i-1} & i > 1 \end{cases}$$

where

θ'_i is a scaled version of θ'_i / Δ*t*, an approximation to the first-order derivative of the steering wheel signal at time step *i*.
Δ*t* is the difference in time between two time steps. It's not needed in order to find the stationary points.

Instead use θ'_i directly, and find all *i* such that either:

$$\theta'_i = 0 \qquad 2 \le i \le T \tag{F1}$$

or:

$$|\, sign(\theta'_i) - sign(\theta'_{i+1}) \,| = 2 \qquad 1 \le i \le T - 1 \tag{F2}$$

where:

$$sign(x) = \begin{cases} -1 & x < 0 \\ 0 & x = 0 \\ 1 & x > 0 \end{cases}$$

Any *i* satisfying equation Equations F1 or F2 is thus a position in the steering wheel angle signal where the approximate first-order derivative of the steering wheel angle is either zero (equation F1), or just about to pass zero (equation F2). Any such point is a stationary point. This procedure is illustrated in Figure F1*.*

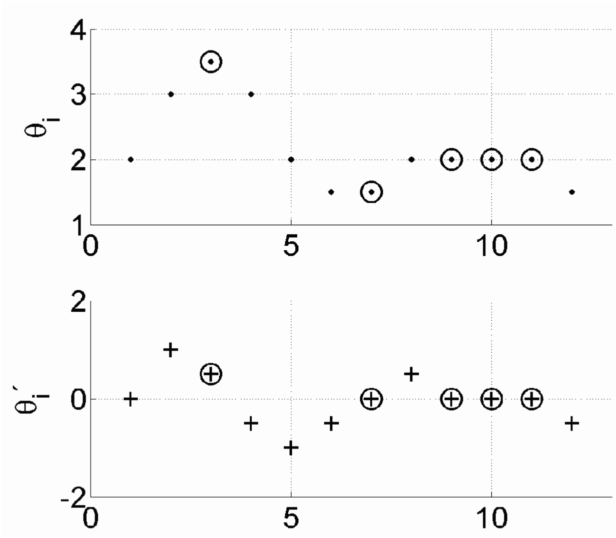

FIGURE F1 - ILLUSTRATION OF THE METHOD FOR FINDING STATIONARY POINTS OF THE STEERING WHEEL ANGLE SIGNAL. An example signal θ_i is plotted in the top graph, and corresponding values of θ'_i are plotted in the bottom graph. i = 3 satisfies equation F2, and i = {7, 9, 10, 11} satisfies equation F1, so i = {3, 7, 9, 10, 11} are stationary points of the steering wheel angle signal.

F.2.3 Identify and count steering wheel reversals

Let *e*(*k*) be the k^{th} value of *i* such that *i* is a stationary point, sorted in time order so that *e*(*k*) > *e*(*l*) if *k* > *l*. For the example of Figure F1, *e*(1) = 3, *e*(2) = 7, *e*(3) = 9, *e*(4) = 10, and *e*(5) = 11. Let *N* be the total number of stationary points. Then the following algorithm counts all upward reversals (from a stationary point of lower angle value to one of higher angle value, e.g. from a local minimum to a local maximum) in the steering wheel angle signal that are bigger than the gap size threshold θ_{min}, which is set to 3 degrees[1].

```
1. $k \leftarrow 0$
2. $N_r \leftarrow 0$
3. For $l$ = [2, 3, 4, …, N]
    a. If $\theta_{e(l)} - \theta_{e(k)} \geq \theta_{\min}$ :
        i. $N_r \leftarrow N_r + 1$
        ii. $R(N_r) \leftarrow [e(k), e(l)]$
        iii. $k \leftarrow l$
    b. Else if $\theta_{e(l)} \leq \theta_{e(k)}$
        i. $k \leftarrow l$
```

This algorithm positions *k* at the first stationary point (*k*=1), and then iterates through the subsequent stationary points until either a stationary point *l* is found that is more than θ_{min} bigger in angle value than the stationary point at *k*, or a stationary point *l* is found that is smaller in angle value than the stationary point at *k*. In the first case an upward reversal has been found. In either case, *k* is set to *l* and the iteration is continued. Setting *k* to *l* in the latter case, when *l* is a stationary point with smaller angle value than *k*, ensures that an upward reversal will be found as soon as possible, since a smaller value of $\theta_{e(k)}$ will be used in step 3.a.

When the algorithm above has terminated, N_r is the number of upward reversals, and *R*(*m*) is a vector with two elements, where the first is the time step where the m^{th} reversal begins, and the second element is the time step where it ends. *R*(*m*) is useful for visualizing the results of the algorithm, as in Figure F1, but if this is not needed step 3.a.ii of the algorithm can be omitted.

---

[1] This calculation uses 3 degrees for the steering reversal threshold. As noted in sections 8.3.1, 8.3.2, and 8.3.3, other threshold values have been used as well.

To count also the downward reversals, the same algorithm is then applied on the negative of the steering wheel angle, $-\theta_i$, instead of on θ_i.

The total number of reversals in the steering wheel angle signal is obtained as the sum of upward and downward reversals.

F.2.4 Calculate Steering Wheel Reversal Rate

The steering wheel reversal rate is calculated as the total number of reversals detected in the steering wheel angle signal, divided by this signal's total length in minutes.

F.3 LIST OF SYMBOLS

Symbol	Definition	Units
Θ_i	value of the low-pass filtered steering wheel angle signal at time step i, with $i = \{1, 2, 3, \ldots, T\}$, where T is the total number of samples in a measurement	deg
θ'_i	scaled version of θ'_i / Δt, an approximation to the first order derivative of the steering wheel signal at time step i	deg/s
θ_{min},	minimum gap size or threshold (set to 3 degrees)	deg
Δt	difference in time between two time steps	s
$e(k)$	k^{th} value of i such that i is a stationary point, sorted in time order so that $e(k) > e(l)$ if $k > l$	

APPENDIX G – CALCULATION OF STEERING ENTROPY (H_P)

G.1 BACKGROUND

While driving a vehicle, drivers continuously assess the situation ahead and unconsciously employ smooth and predictable steering control. Smooth in this instance can be defined as turning the steering wheel a little at a time in small increments. When drivers are distracted (or impaired), the driver does not monitor the roadway environment as effectively, leading to greater lateral lane deviations and more large amplitude steering corrections. The steering entropy method is connected to these corrective steering maneuvers. Steering predictability decreases as drivers introduce larger error-correcting maneuvers. Drivers introduce more error-correcting maneuvers as distraction (due to a secondary task or impairment) increases.

Steering wheel entropy, as defined in section 4, is a measure of the consistency/randomness of the steering angle computed by using a series of previous steering wheel angles to calculate a subsequent steering angle.

G.2 1999 COMPUTATIONAL METHOD

This appendix is excerpted from Nakayama, Futami, Nakamura, and Boer (1999) and from Kersloot, Flint, and Parkes (2003). Be sure to collect baseline driving data, i.e. normal driving without some added task(s) of interest. The information of interest is the difference between the baseline and the dual task conditions.

G.2.1 Sample and Record the Steering Wheel Angle every 15-50 ms

The exact sampling frequency is not critical except that very high sampling frequencies result in steering angle prediction errors that are too noisy, causing a loss of sensitivity in Hp.

NOTE: Kersloot, Flint, and Parkes (2003) sampled once every 15 ms; Nakayama et al. (1999) used 50 ms.

G.2.2 Re-sample the Steering Wheel Angle at 150 ms Time Steps (~7 Hz)

This was indicated to be a good time step interval in previous research. It also corresponds to the lowest sampling frequency that can be used to represent a human operator's control response in manual tracking tasks (Nakayama, et. al. 1999).

G.2.3 Compute a Second-Order Taylor Expansion for Predicted Steering Angle, $\theta_p(n)$, at a given time using the Previous Three Time Steps $\theta(n-1)$, $\theta(n-2)$, and $\theta(n-3)$.

$$\theta_p(n) = \theta(n-1) + [\theta(n-1)-\theta(n-2)] + \tfrac{1}{2} [(\theta(n-1)-\theta(n-2)) - (\theta(n-2)-\theta(n-3))]$$

$$\rightarrow \theta_p(n) = 5/2\ \theta(n-1) - 2\ \theta(n-2) + 1/2\ \theta(n-3)$$

where $\theta_p(n)$ is the predicted steering angle at time n
$\theta(n)$ is the actual steering angle at time n

G.2.4 Compute the Prediction Error and Verify Normality of its Frequency Distribution

As shown in Figure G1, the steering wheel angle prediction error e(n) is

$$e(n) = \theta_p(n) - \theta(n)$$

A sample frequency distribution is plotted in Figure G2. Check that e(n) is normally distributed. Then, compute the mean and standard deviation of e(n).

G.2.5 Determine Parameter α for Normal (baseline) Driving

Determine a parameter α from the prediction error distribution for the baseline condition so that 90 percent of the data (1.645 standard deviations) falls between $-\alpha$ and α (Figure G2).

The shape of the prediction error distribution becomes narrower (smaller α) as the driver's steering behavior becomes smoother. The value of α indicates the fundamental steering behavior of an individual in the baseline reference condition.

It is used as the reference to compare measurements made of the workload incurred by different activities. Nakayama, Futami, Nakamura, and Boer (1999) reported α = 1.52 averaged across participants for normal driving. Kersloot, Flint, and Parkes (2003) reported α = 1.41 for 6 participants for normal driving on 1 km of straight road.

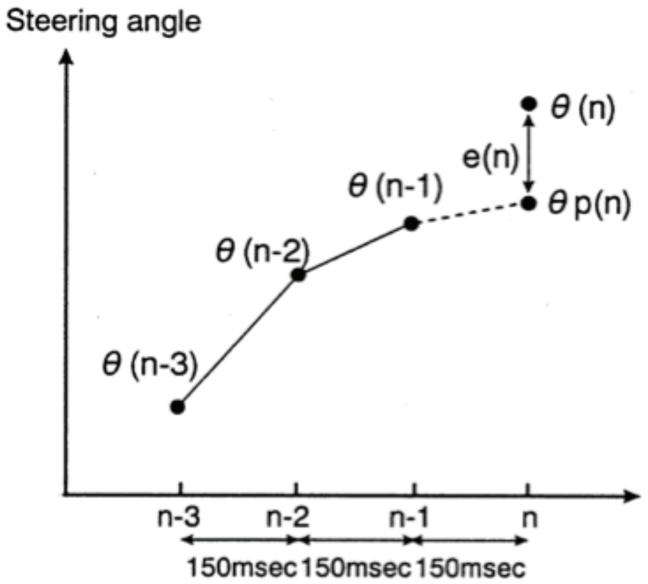

FIGURE G1 - COMPUTATION OF STEERING ANGLE ERROR

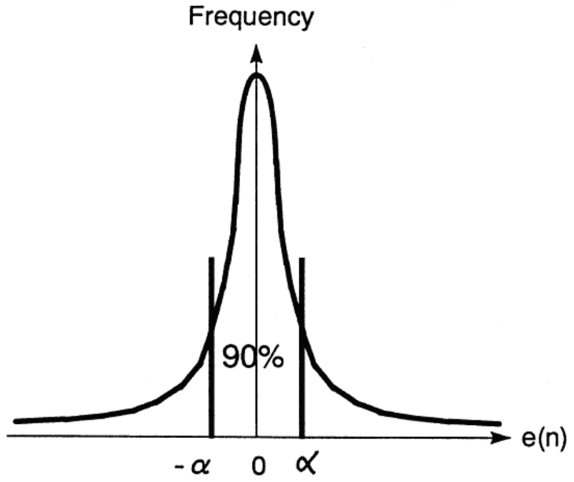

FIGURE G2 -- FREQUENCY DISTRIBUTION OF e(n). DETERMINE α AS SHOWN.

G.2.6 Partition the Frequency Distribution of e(n) into 9 Bins as shown in Figure G3

For each participant partition their predicted steering error frequency distribution into 9 bins based on each individual's α value for normal driving (baseline condition).

Determine the number of prediction errors that fall into each bin and compute the relative frequency (probability, p_i) for each bin.

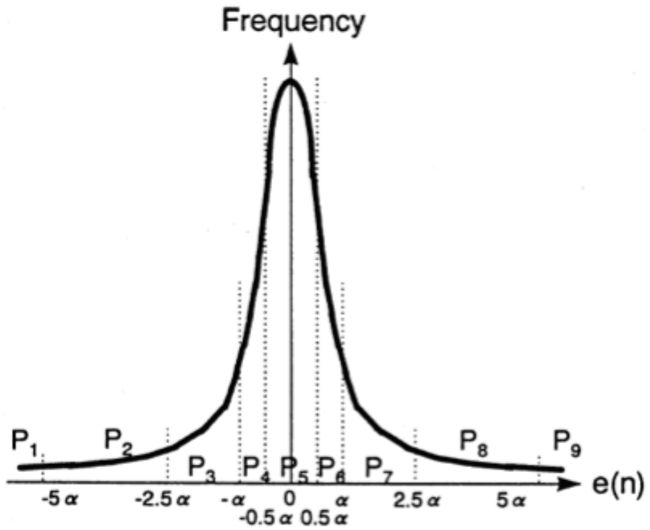

FIGURE G3 - BINNING THE FREQUENCY DISTRIBUTION OF e(n)

G.2.7 Compute the Steering Entropy, Hp

$$Hp = \sum_{i=1}^{9} - p_i Log_9 p_i$$

where p_i is the probability of being in bin i $i = 1,\ldots, 9$

Using log base 9 assures entropy values between zero and one when participants perform different activities while driving the same course. As shown in Figure G4, increasing driver workload changes the predicted error frequency distribution and the corresponding steering entropy.

NOTE: The units of Hp are 9-ary units, since the logarithm is base 9. When log base 2 is used, the units of entropy are information bits. It’s acceptable not to report the units for steering entropy when log base 2 is not used.

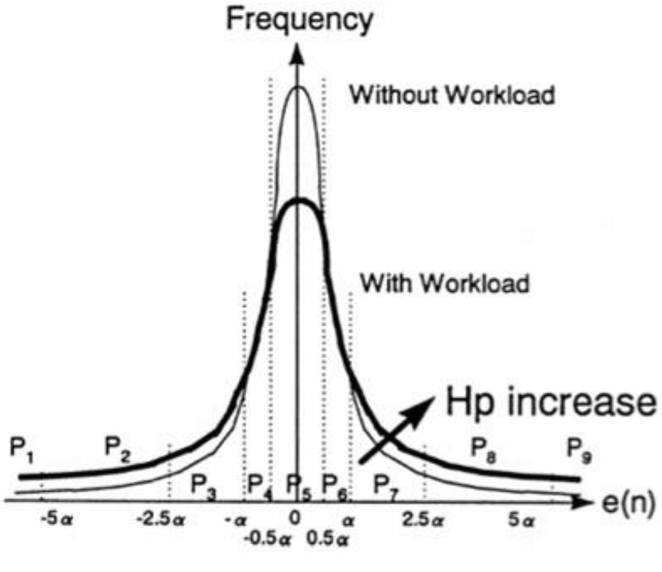

FIGURE G4 - CHANGE IN FREQUENCY DISTRIBUTION OF e(n) WITH WORKLOAD

G.3 2005 COMPUTATIONAL METHOD

Boer, Rakauskas, Ward, and Goodrich, (2005) modified the original (1999) steering entropy computational procedure to increase its sensitivity to detect changes in steering behavior. The original SE method attributed its sensitivity primarily to high frequency corrective maneuvers. These corrective steering actions do indeed account for a small increase in the high frequency power (as observed in all subjects), however, the steering maneuver following these initial jerky corrective responses is of lower frequency and much longer duration. Thus, an increase in low frequency power is also expected.

(When examining the power spectral density of steering behavior for a cell phone task, Boer observed that some subjects coped with the additional task by using a steering behavior with an increased power at low frequencies, some primarily increased power at higher frequencies, and some increased overall power at all frequencies.) This is one of the reasons why the optimized SE algorithm yields superior sensitivity using a lower re-sampling frequency and a prediction filter with less low frequency attenuation than the original Taylor series, thereby increasing sensitivity to low frequency changes while maintaining sensitivity to high frequency changes in steering behavior.

Using the 2005 computational method for determining steering entropy both the high frequency quicker steering corrections and low frequency slower steering corrections are captured. Accounting for both types of corrections provides a more complete and sensitive picture of the steering behavior.

After the steering angle data is collected (see G.2.1), perform the following steps:

G.3.1 Filter the steering data with a 5th order low-pass Butterworth filter

The filter cutoff frequency is 3/7 of the sample rate in G.2.1.

G.3.2 Re-sample the steering data every 4 Hz or 250 ms

The 1999 SE algorithm was based on a Taylor expansion to generate prediction errors from a steering profile re-sampled every150 ms, i.e. sampled down to about 7Hz. Boer (2005) determined that a re-sampling frequency of 4 Hz gave the most sensitive and robust assessment of steering behavior changes for both the AR (see G.3.3) and Taylor filters.

The optimality of a 4Hz re-sampling frequency is attributed to the fact that the resulting frequency range (0-2 Hz) spans drivers natural frequency range. A very low re-sampling frequency eliminates high frequency effects from the analysis (some subjects only show a signature effect at high frequencies). On the other hand, a very high re-sampling focuses the AR-model too much on high frequencies that are beyond the natural operating range of the human driver, thereby yielding an overall reduction in sensitivity; furthermore, the power spectral densities typically show very little differences at the highest frequencies thus yielding a predicted error distribution that focuses too much on the noise.

G.3.3 For each participant compute steering prediction error, e_n, for the baseline reference condition using a moving average (MA) filter derived from a 3rd order autoregressive model of steering data.

$$e_n = \theta_n + a_1\theta_{n-1} + a_2\theta_{n-2} + a_3\theta_{n-3}$$

Each subject has unique prediction error filter. The a_i coefficients are determined from the third-order autoregressive (AR) model of the steering time series, θ_n, assuming a Gaussian (normal or white noise) error distribution.

$$\theta_n = - a_1\theta_{n-1} - a_2\theta_{n-2} - a_3\theta_{n-3} + e_n$$

This can be done in Matlab using the Burg algorithm (“arburg”). The Taylor expansion (1999 method) and the AR-model derived MA filter are both high-pass filters (Figure G5). Note that the AR-derived filter method weights low frequencies more than Taylor, which greatly attenuates the low frequencies.

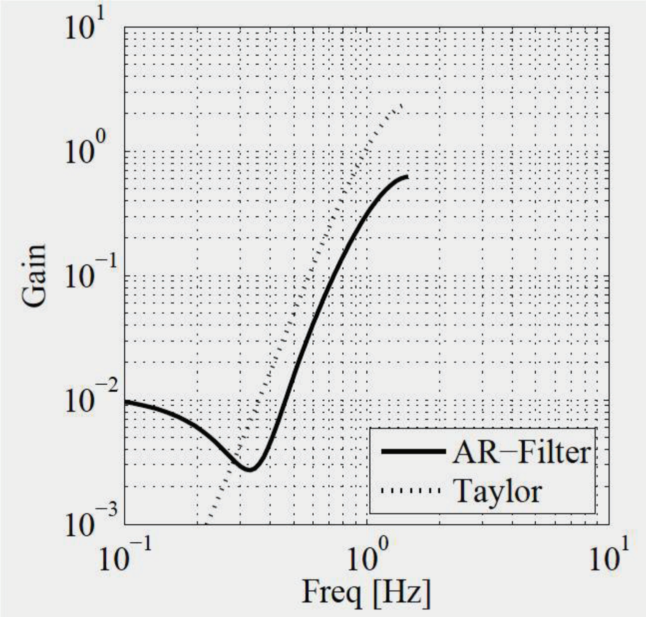

FIGURE G5 - COMPARISON OF AR AND TAYLOR HIGH-PASS FILTERS

The advantage of using an AR model is that it is optimally tuned to the frequencies present in the baseline steering data, which depends on a number of factors such as driver skill, road curvature, speed, lateral disturbances, vehicle dynamics, etc. Because of the optimally frequency-tuned AR model, a steering entropy method based on AR-models is also optimally sensitive to deviations from this baseline steering spectrum.

The AR-model coefficients are estimated on the first 60 s of the baseline data (reference). The AR-model derived MA filter is estimated from each subject's baseline reference data separately and used only on that subject's unused baseline data and its condition trials to assess the shift in predicted steering error distributions from baseline to condition. (This differs from using all available baseline reference data to estimate a common baseline prediction filter.)

G.3.4 Compute predicted steering errors for the baseline and task conditions using a_i coefficients from the baseline reference.

The a_i coefficients from the baseline reference prediction error filter are used to generate the baseline predicted steering error as well as the task condition(s) predicted steering errors:

$$pe_n^{bas_m^{ref}} = s_n^{bas_m^{ref}} + a_1^{bas_m^{ref}} s_{n-1}^{bas_m^{ref}} + a_2^{bas_m^{ref}} s_{n-2}^{bas_m^{ref}} + a_3^{bas_m^{ref}} s_{n-3}^{bas_m^{ref}}$$

$$pe_n^{bas_m} = s_n^{bas_m} + a_1^{bas_m^{ref}} s_{n-1}^{bas_m} + a_2^{bas_m^{ref}} s_{n-2}^{bas_m} + a_3^{bas_m^{ref}} s_{n-3}^{bas_m}$$

$$pe_n^{cond_m} = s_n^{cond_m} + a_1^{bas_m^{ref}} s_{n-1}^{cond_m} + a_2^{bas_m^{ref}} s_{n-2}^{cond_m} + a_3^{bas_m^{ref}} s_{n-3}^{cond_m}$$

where

subscript *m* denotes subject m,
S_n is the steering angle at time step n,
superscript bas_m^{ref} refers to the first 60 seconds of subject *m*'s baseline data,
bas_m refers to the second 60 seconds of the baseline data, and
$pe_n^{cond_m}$ is the predicted steering error for subject m for a given test condition, *cond.*

G.3.5 Compute predicted steering error frequency distributions for baseline reference, baseline, and task conditions

G.3.6 Determine Parameter α for Normal (baseline = reference) Driving

To measure the degree to which the predicted error distribution widens under different conditions, the probability density function of the baseline reference predicted errors is used. The parameter alpha is a measure of the width of the predicted steering error frequency distribution for baseline (reference) driving. (An alpha value could also be calculated as a measure of the degree to which the predicted error distribution widens under different conditions than baseline.)

Determine parameter α from the prediction error distribution for the baseline condition so that 60 percent of the data (0.843 standard deviations) falls between $-\alpha$ and α. Boer (2005) determined that this alpha value gave the most sensitive and robust assessment of steering behavior changes.

G.3.7 Partition the Frequency Distributions of e(n) into 14 Bins

Boer found that using 4-10 bins yields fewer significant results (i.e., the algorithm is less sensitive), but with 12 to 32 bins the same results are obtained. He recommended that the number of bins is set at K=14, to avoid that small data sets results in too many empty bins.

The predicted error distribution is approximated with 14 bins whose bin boundaries are

$$-10e12, -6\alpha, -5\alpha, -4\alpha, -3\alpha, -2\alpha, -\alpha, 0, \alpha, 2\alpha, 3\alpha, 4\alpha, 5\alpha, 6\alpha, 10e12$$

where the extreme range of *-10e12* and *10e12* assure that all predicted errors are captured.

Calculate the probability that a reference baseline predicted error falls in a bin. This is achieved by simply counting what fraction of reference predicted errors falls in a bin. This may leave some of the outer bins empty. To avoid extremely low probabilities and thus extremely high "entropies," all bins with probabilities less than 1.0e-3 are replaced by 1.0e-3 (this is necessary to avoid that some prediction errors from non-reference baseline or condition data receive an excessively high sample entropy, thus assuring that the method does not become extremely sensitive to one or two outliers but requires a reasonable number of high prediction errors to substantially increase the entropy of the condition under investigation).

G.3.8 Compute the Steering Entropy, H

The entropy calculation of the binned predicted error distribution assigns high weight to outliers (i.e., those prediction errors that fall in low probability bins). The entropy of subject *m*'s second half baseline data as well as the various task conditions are computed using:

$$H^{cond_m} = \frac{\sum_{n=1}^{N^{cond_m}} -\log_2\left(P^{bas_m^{ref}}\left(pe_n^{cond_m}\right)\right)}{N^{cond_m}} = \sum_{k=1}^{K}\left\{-\frac{N_k^{cond_m}}{N^{cond_m}}\log_2\left(P_k^{bas_m^{ref}}\right)\right\} = \sum_{k=1}^{K}\left\{-P_k^{cond_m}\log_2\left(P_k^{bas_m^{ref}}\right)\right\}$$

where

H^{cond_m} is the steering entropy H for subject m for a given task condition,

K is the number of bins (14 bins are recommended),

$P_k^{bas_m^{ref}}$ is the probability associated with subject *m*'s bin *k,* and

$N_k^{cond_m}$ is the number of task condition predicted errors from subject *m* that fall in bin *k* of subject *m*'s discretized reference-baseline predicted error distribution.

This equation is slightly different from the original one to yield higher sensitivity. The old equation only uses the bins from the baseline reference but not the baseline reference probabilities as argument to the 2 log function. Note that log base 2 is used, so the units of entropy are bits.

G.4 MATLAB PROCEDURE TO COMPUTE STEERING ENTROPY

The free parameters of the new SE algorithm are the re-sampling frequency, the alpha value, the number of bins, and whether a Taylor expansion or an AR-model-based prediction filter is used. Alpha in the case is the proportion of steering errors in each tail of the predicted error distribution (not the predicted error cutoff value shown in Figure G2). For the 1999 model alpha = 0.05; for the 2005 model, alpha = 0.2 is recommended. Other values of alpha can be tried in an attempt to achieve more significant results. Choosing a different alpha value corresponds to encompassing 100(1 – 2 alpha) percent of the predicted errors between the – α and α boundaries in Figure G2.

G.5 LIST OF SYMBOLS

Symbol	Definition	Units
$\theta_p(n)$	predicted steering wheel angle at time step n	deg
$\theta(n)$ or s_n	actual steering wheel angle at time step n	deg
$e(n)$ or pe_n	prediction error for steering wheel angle at time step n	deg
α	value that determines bin boundaries of predicted steering error frequency distribution	deg
a_i	i^{th} coefficient of the autoregressive model	
K	number of bins	
P_k	probability of predicted error being in bin k	
Hp or H	steering entropy	bits if log base 2 is used

APPENDIX H – CALCULATION OF TIME TO LINE CROSSING, TRIGONOMETRIC METHOD

H.1 BACKGROUND

Time to Line Crossing (TLC) as defined in section 9.4.1 is the time taken for a vehicle to reach a lane boundary, either the inside edge of the lane marking (Option A), the centerline (Option B), or the outside edge (Option C).

TLC can only be computed when there are well-defined lane markings. TLC measures are most commonly reported for driving simulator studies, for which the center of each edge marking and the mirror-to-mirror vehicle width are readily available. In real vehicles, time-to-line crossing is often determined using data from a camera near the center of the mirror (which provides distance to the lane edge and vehicle yaw angle) and data from acceleration sensors. The trigonometric method given in this appendix is the preferred method for calculating TLC in simulator studies rather than using the two common approximations: (1) dividing lateral position by lateral velocity, and (2) dividing lateral position by the sum of lateral velocity and lateral acceleration. Approximation (2) is used for field experiment data. Poor quality lateral position data will create worse TLC data regardless of the method used to compute TLC.

H.2 COMPUTATIONAL METHOD

The estimation procedures given below for straight and curved roads are consistent with those described by van Winsum, Brookhuis, and de Waard (2000), pages 48-49.

An exact trigonometric computation, sometimes abbreviated as TLC_tri, is given by:

TLC_tri = DLC/u for u>0, else TLC_tri = infinity

where: DLC is the distance to line crossing along vehicle path (m)
u is the vehicle speed (m/s)

H.2.1 TLC for straight roads

Referring to Figure H1, the distance to line crossing for a straight road is given by

$DLC = \alpha * R_v$

where α is the angle subtended by the path of the vehicle from its initial position to the road line marking
R_v is radius of the curved vehicle path

Also

$\alpha = \arccos((A^2 + R_v^2 - C^2)/(2 * A * C))$

$R_v = u/r$

and

$A = R_v - A'$

$A' = y/\cos(\alpha_1)$

$C = [2 * A * \cos(\beta) + \mathrm{SQRT}\{(2 * A * \cos(\beta))^2 - 4 * (A^2 - R_v^2)\}] / 2$

where

r is the vehicle yaw rate (radians/s)

y is the distance between front wheel and lane boundary (perpendicular to the road)

α_1 is the angle between the line perpendicular on the road and the line from the front where to the center point (X_v, Y_v) of the vehicle's curved path

C is the longitudinal distance of vehicle travel on the roadway

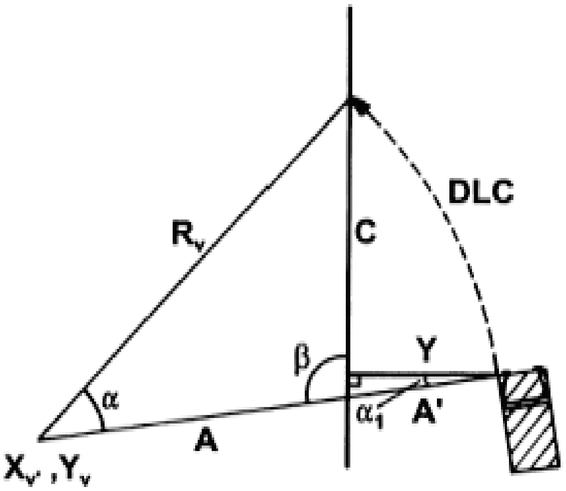

FIGURE H1 - STRAIGHT ROAD TLC PARAMETERS FOR DETERMINING THE LENGTH OF THE ARC DLC

H.2.2 TLC For Curved Roads

Referring to Figure H2, the distance to line crossing DLC for a curved road is also given by

$$DLC = \alpha * R_v$$

Angle α is given by

$$\alpha = \beta - \alpha_1$$

$$\alpha_1 = \arccos((A^2 + R_r^2 - R_v^2) / (2 * A * R_v)$$

where

β = angular difference between the line from the center point of the vehicle curve (X_v, Y_v) to the center point of the road curve (X_r, Y_r) and the line from the center point of the vehicle curve (X_v, Y_v) to the left front wheel (if the vehicle turns toward the inner lane boundary)

A = distance between the center point of the road curve and center point of the vehicle curve

R_r = radius of the curved road segment

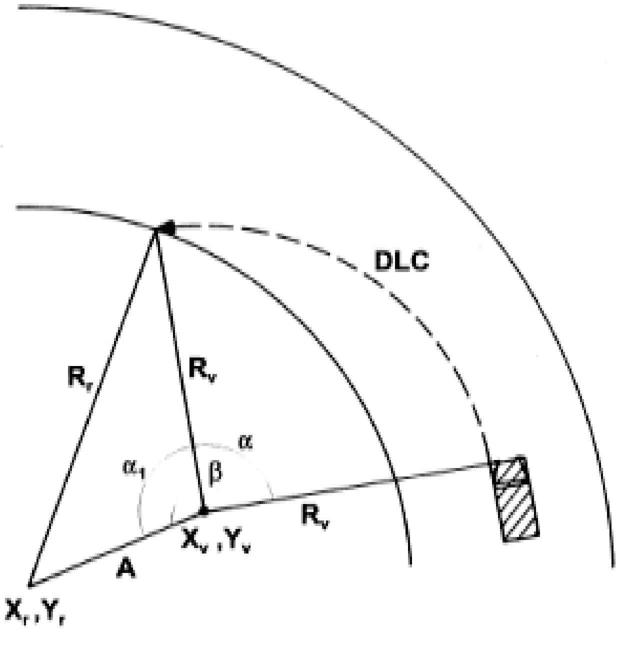

FIGURE H2 - CURVED ROAD TLC PARAMETERS

H.3 LIST OF SYMBOLS

Symbol	Definition	Units
TLC	Time to Line Crossing	s
TLC_tri	TLC computed by an exact trigonometric computation	s
u	vehicle speed	m/s
DLC	distance to line crossing along the vehicle path	m
R_v	radius of the curved vehicle path $R_v = u/r$	m
α	angle subtended by the path of the vehicle from its initial position to the road line marking	radians
α_1	angle between the line perpendicular on the road and the line from the front to the center point (X_v, Y_v) of the vehicle's curved path	radians
β	angular difference between the line from the center point of the vehicle curve (X_v, Y_v) to the center point of the road curve (X_r, Y_r) and the line from the center point of the vehicle curve (X_v, Y_v) to the left front wheel (if the vehicle turns toward the inner lane boundary)	radians
A	distance between the center point of the road curve and center point of the vehicle curve	m
r	vehicle yaw rate	radians/s
y	distance between front wheel and lane boundary (perpendicular to the road)	m
C	longitudinal distance of vehicle travel on the straight roadway	m

APPENDIX I – CALCULATION OF MINIMUM TIME TO LINE CROSSING - APPROXIMATE METHOD FOR FIELD DATA

I.1 BACKGROUND

As noted in Appendix A**,** Time to Line Crossing (TLC) as defined in section 9.4.1 is the time taken for a vehicle to reach a lane boundary, either the inside edge of the lane marking (Option A), the centerline (Options B), or the outside edge (Option C). In some on-road tests all of the data needed for precise estimates of TLC are not available. In such instances the estimates described in this appendix should be used.

I.2 COMPUTATIONAL METHOD

The method given below for calculating minimum TLC from field data was taken almost verbatim from the AIDE report (Ostland, et.al., 2005, page 148). AIDE derived it from van Winsum and Godthelp (1996).

I.2.1 Approximations for Determining TLC

Approximations are often used to calculate TLC (van Winsum & Godthelp, 1996), since they require fewer parameters than true TLC and also can be applied on field experiment data. Calculation of approximate TLC is done using lateral position data.

I.2.2 Computations

The minimum time to line crossing is

TLC = LP_right/(LV+LA) if LA<0 (accelerating to the right)

TLC = LP_left/(LV+LA) if LA>0 (accelerating to the left)

TLC is undefined for the following conditions:

LA=0
LP_right < 0 or LP_left < 0 (outside the lane)
TLC > 20 seconds (proposed and used in the HASTE project (Östlund et. al., HASTE Deliverable 2, 2004)

where

LP_right is the lateral distance from right wheel to right lane marking

LP_left is the lateral distance from left wheel to left lane marking

LV is the vehicle lateral velocity relative to the road

LA is the vehicle lateral acceleration relative to the road (i.e. the rate of change in lateral velocity)

A sample TLC waveform is shown in Figure I1. TLC calculated for the right side is negative, and for the left side positive. When determining local minima, only use TLC waveforms that are at least *1 s long* (proposed and used in HASTE). For these, identify the local minima (maxima for negative waveforms), as shown in Figure I1.
Record the minimum TLC values (TLC_{min}) that have durations of at least 1 s.

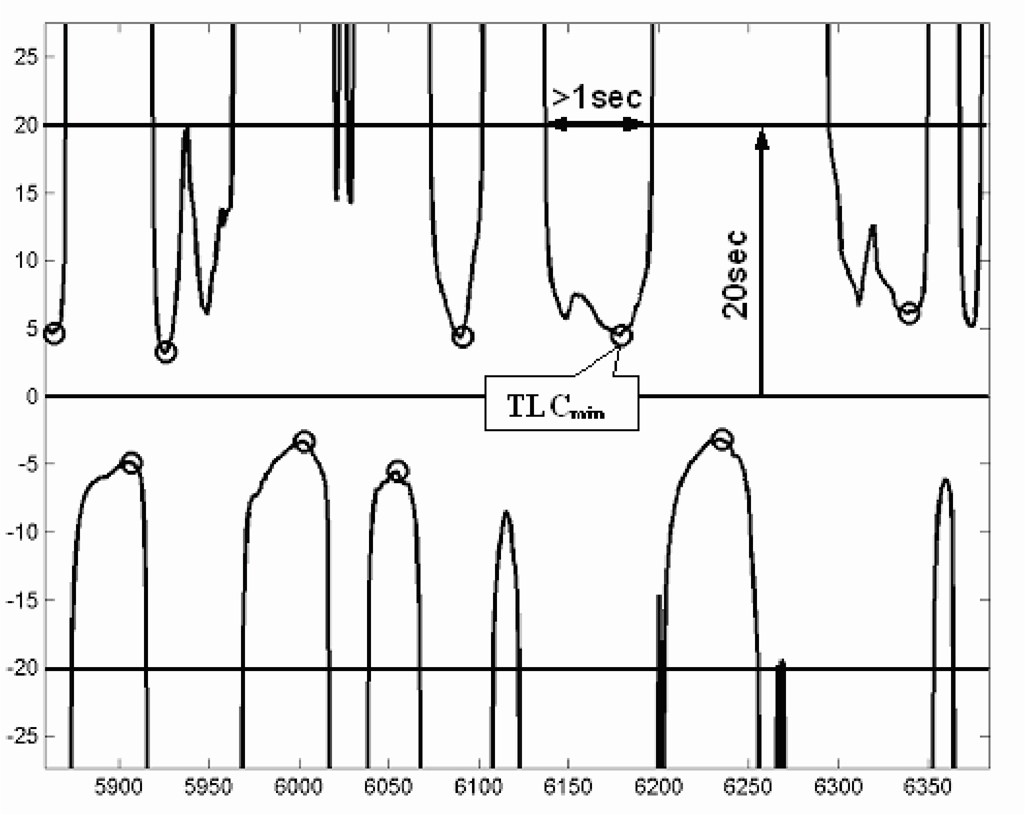

FIGURE I1--TLC WAVEFORMS AND IDENTIFICATION OF TLC MINIMUM VALUES.

Source: HASTE Deliverable 2 (Östlund et al, 2004)

I.2.3 Low-Pass Filtering and Data Quality

If TLC is computed from on-the-road data, some filtering of the lateral position, lateral velocity and lateral acceleration data is needed to avoid amplification of noise in those measures. That noise will be apparent if a visual inspection of the data shows large changes in the value of each measure that then a return to approximately the prior value. Stable TLC data will resemble Figure I1. A low-pass filter with cutoff frequency no less than 3 Hz is a good starting point for stabilizing the values of lateral position, lateral velocity, and lateral acceleration. Poor precision in the lateral position data cannot, however, be fully compensated by filtering. Make a competent assessment of the usability of your TLC data.

I.3 LIST OF SYMBOLS

Symbol	Definition	Units
TLC	Time-To-Line-Crossing	s
TLC_{min}	minimum TLC values	s
LP_right	lateral distance from right wheel to right lane marking	m
LP_left	lateral distance from left wheel to left lane marking	m
LV	vehicle lateral velocity relative to the road	m/s
LA	vehicle lateral acceleration relative to the road (i.e. the rate of change in lateral velocity)	m/s^2

NOTE: The distance from the wheel to the lane marking could be to the inside, center, or outside of the marking and must be specified.

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